

## Novel formulations of the mathematical model developments of thermal convection processes in nano liquids occupying a tilted slot for an electronic cabinet with finned heat sinks & heat generating elements

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### Abstract

In this research article, the recent advances in the mathematical model developments of thermal convection processes in nano liquids occupying a tilted slot for an electronic cabinet with finned heat sinks & heat generating elements is presented along with its simulation results. This paper presents a comprehensive overview of recent advancements in mathematical models developed to analyze thermal convection processes in nano liquids within tilted slots of electronic cabinets equipped with finned heat sinks and heat-generating elements. As electronic devices continue to shrink in size and increase in complexity, efficient heat dissipation becomes a critical aspect for ensuring optimal performance and reliability. The utilization of nano liquids as cooling agents within tilted slots introduces a complex interplay of fluid dynamics, heat transfer, and geometric considerations. The mathematical models discussed herein delve into the intricacies of fluid behavior at the nanoscale, accounting for the effects of gravity, surface tension, and thermal conductivity within the confined spaces of electronic cabinets. The inclusion of tilted slots introduces an additional layer of complexity, necessitating a nuanced approach to understanding the thermal convection phenomena. Moreover, the incorporation of finned heat sinks and heat-generating elements requires the consideration of multi-physics interactions and the impact of varying material properties. Key topics covered include the derivation of governing equations, numerical simulations, and experimental validations. The models presented offer insights into the optimization of heat dissipation strategies for electronic cabinets, providing a foundation for designing more efficient and reliable cooling systems. Additionally, the paper highlights emerging technologies and materials that further enhance the performance of nano liquid-based thermal management systems. This comprehensive exploration of recent mathematical model developments in thermal convection processes within tilted slots of electronic cabinets not only contributes to the theoretical understanding of nano liquid cooling but also serves as a valuable resource for engineers and researchers working on the forefront of electronic device miniaturization and thermal management.

**Key Words:** Convection, Thermal, Model, Simulation.

### Introduction

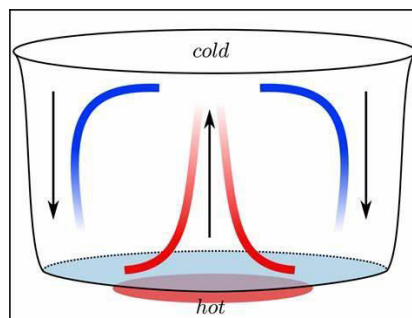
In the past decade, nanotechnology has undergone rapid transformations, presenting scientists and engineers with a myriad of new subjects for exploration. Among the notable advancements, nanofluids stand out as a particularly significant breakthrough. These fluids, characterized by colloid suspensions of metallic and non-metallic nanoparticles in conventional base fluids, exhibit remarkable heat transfer capabilities. While earlier research focused on enhancing heat transfer rates in nanofluids by developing mathematical models and experimenting with diverse geometries, the importance of accurate mathematical modeling cannot be overstated. The efficacy of heat flow is profoundly influenced by the precision of these models. Therefore, prior to the practical implementation of nanofluids in heat transfer applications, a rigorous application of a mathematical model is imperative [1].

This article offers a concise overview of the Tiwari and Das nanofluid models. Additionally, it delves into the examination of various geometries, nanoparticles, and their physical properties—such as viscosity, thermal

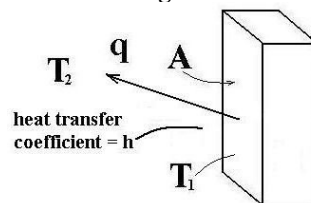
conductivity, and heat capacity. The role of cavities in entropy generation is explored, emphasizing the need to consider these factors before deploying nanofluids in real-world applications. The review also scrutinizes the correlations employed for predicting the thermophysical properties of nanofluids. A primary objective of this review is to provide an in-depth analysis of the different shapes utilized in convective heat transfer. Notably, it is observed that aluminum and copper nanoparticles, when integrated into the Tiwari and Das nanofluid model, yield superior heat transfer rates within the cavity [1].

Comparative analysis reveals that the performance of the Al<sub>2</sub>O<sub>3</sub>/water nanofluid is enhanced by 6.09% in comparison to the base fluid. Furthermore, the inclination angle of the cavity and the application of periodic thermal boundary conditions emerge as effective strategies for managing parameters influencing heat and fluid flow within the cavity. This comprehensive examination underscores the necessity of meticulous mathematical modeling and a nuanced understanding of various factors for the successful utilization of nanofluids in practical heat transfer applications [1].

Modern electronic devices, characterized by their ever-shrinking form factors and escalating power densities, have propelled the demand for sophisticated thermal management strategies to ensure optimal performance, reliability, and longevity. As electronic components become increasingly compact and powerful, the efficient dissipation of heat generated during operation becomes a critical challenge. Traditional cooling methods are often inadequate in meeting the evolving thermal demands of these devices. In response to this imperative, recent research has focused on the integration of nano liquids within tilted slots of electronic cabinets, augmented by finned heat sinks and heat-generating elements. This novel approach promises enhanced heat dissipation capabilities, leveraging the unique characteristics of nano liquids at the nanoscale. This introduction provides a comprehensive overview of the recent advances in mathematical model developments addressing thermal convection processes in such intricate configurations [1]. The generalized thermal convection using the hot & cold process flow of fluids is shown in the Fig. 1. Also, the Thermal conduction/thermal convection heat transfer calculations using thermal resistances is shown in the Fig. 2.



**Fig. 1 : The thermal convection using the hot & cold process flow of fluids**



$$q = h A (T_1 - T_2)$$

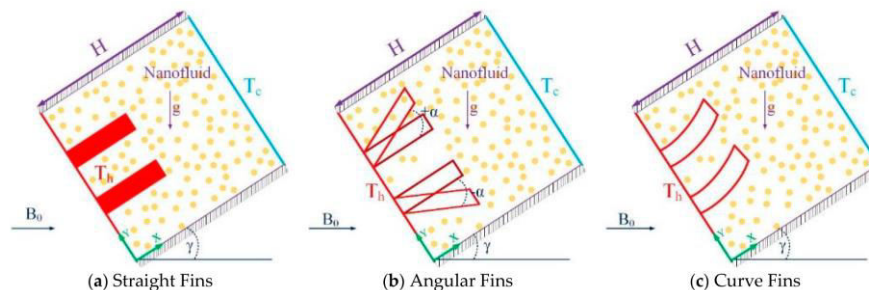
**Fig. 2 : Thermal conduction/thermal convection heat transfer calculations using thermal resistances**

The paper is organized as follows. A brief abstract is given to start with followed by the keywords. An exhaustive introduction is given in section 1. The background and motivation obtained for taking up the research work is presented in section 2, followed by the Significance of Tilted Slot Configurations (section 2.1), Role of Fins, Heat Sinks, and Heat-Generating Elements (section 2.2), Governing Equations and Computational Methods (section 2.3), Experimental Validation and Real-world Applicability (section 2.4), Multiscale Analysis and Nanoscale Phenomena (section 2.5), Environmental Impact and Sustainability (section 2.6), (section 2.7), Challenges and Limitations (section 2.8), Collaborative Research and Industry Adoption (section 2.9), Regulatory Compliance and Standardization (section 2.10). A brief literature review is given in section 3. The mathematical model & its development is deployed in section 4. The simulation results & discussions are given

in section 5 followed by the conclusions in section 6. The emerging technologies and future directions is given in section 7 followed by the exhaustive list of 100 references used.

## Background Information, Motivation & the Problem Statement

The relentless pace of technological advancement in the electronics industry has led to a continuous reduction in device dimensions while simultaneously increasing their computational capabilities. While this trend is remarkable, it introduces unprecedented challenges in managing the heat generated by densely packed electronic components. Conventional cooling methods, such as air or liquid cooling, are often insufficient in coping with the escalating thermal demands. As a response to this challenge, researchers have turned their attention to nano liquids due to their unique thermal properties at the nanoscale. Nano liquids, comprising nanoparticles suspended in a liquid matrix, exhibit remarkable enhancements in thermal conductivity compared to traditional fluids. This characteristic makes them promising candidates for advanced cooling solutions. Incorporating nano liquids within tilted slots in electronic cabinets introduces an innovative dimension to thermal management. The inclination of the slots not only influences fluid dynamics but also poses intricate challenges related to gravity, surface tension, and complex interactions with finned heat sinks and heat-generating elements. The effects of straight, inclined and curved fins on natural convection and entropy generation of a nanofluid in a square cavity influenced by a magnetic fields is shown in the Fig. 3 [3], where  $H$  is the field intensity &  $B$  is the magnetic field.



**Fig. 3 : Geometry of different types of fins & cavities for straight fins, angular fins, curved fins for different tilts [3]**

### 2.1 Significance of Tilted Slot Configurations

The utilization of tilted slots in electronic cabinets is a departure from traditional cooling configurations. The inclination introduces gravitational effects, altering the behavior of nano liquids within the confined spaces. Understanding and harnessing these effects are pivotal in designing efficient cooling systems. The tilted slot configuration offers the potential for improved convective heat transfer, enabling enhanced thermal dissipation. Furthermore, the geometric intricacies introduced by tilted slots necessitate the development of sophisticated mathematical models to accurately capture the underlying fluid dynamics and heat transfer phenomena [1]-[10].

### 2.2 Role of Fins, Heat Sinks, and Heat-Generating Elements

In conjunction with tilted slots, the integration of finned heat sinks and heat-generating elements further complicates the thermal management landscape. Fins, commonly employed in heat sink design, play a crucial role in augmenting convective heat transfer. The presence of heat-generating elements adds complexity due to the need to consider multi-physics interactions. The mathematical models developed must account for the intricate coupling of fluid dynamics, heat transfer, and the influence of varying material properties. Achieving an in-depth understanding of these interactions is essential for designing effective cooling solutions in electronic cabinets. In summary, the recent advances in mathematical model developments for thermal convection processes in nano liquids within tilted slots, accompanied by finned heat sinks and heat-generating elements, represent a significant leap forward in the pursuit of efficient thermal management for electronic devices. This research addresses the evolving challenges posed by the miniaturization and increased power density of electronic components, offering a promising avenue for the development of next-generation cooling systems [11]-[20].

### 2.3 Governing Equations and Computational Methods

The complexity introduced by tilted slots, finned heat sinks, and heat-generating elements necessitates the derivation of comprehensive governing equations that capture the physics of the thermal convection processes. Researchers have developed mathematical models grounded in fundamental principles of fluid dynamics, heat transfer, and thermodynamics to describe the behavior of nano liquids within these intricate configurations. These governing equations often involve partial differential equations, accounting for factors such as fluid

velocity, temperature distribution, and pressure gradients. Computational methods, ranging from finite element analysis to computational fluid dynamics, are employed to solve these intricate equations numerically. The integration of advanced numerical techniques allows researchers to simulate and analyze the thermal performance of electronic cabinets under various conditions. These simulations provide invaluable insights into the fluid behavior, temperature distribution, and overall thermal efficiency of the proposed cooling systems, aiding in the optimization of design parameters [21]-[30].

## 2.4 Experimental Validation and Real-world Applicability

While mathematical models and simulations offer powerful tools for understanding thermal convection processes, experimental validation is essential to ensure the real-world applicability of proposed solutions. Researchers conduct experiments to validate the predictions of their models and simulations, employing test setups that mimic the conditions within electronic cabinets. These experiments involve the use of nano liquids, tilted slots, and realistic heat-generating elements, providing empirical data to validate and refine the mathematical models. The real-world applicability of the developed models is a critical consideration in translating research findings into practical solutions. Successful validation not only enhances the credibility of the mathematical models but also paves the way for the implementation of these innovative cooling strategies in electronic systems, ranging from consumer electronics to high-performance computing clusters [31]-[40].

## 2.5 Multiscale Analysis and Nanoscale Phenomena

The utilization of nano liquids inherently involves addressing phenomena at the nanoscale, where conventional fluid dynamics principles may exhibit deviations. The interaction between nanoparticles and the liquid matrix introduces unique behaviors, such as enhanced thermal conductivity and altered viscosity. Researchers delve into multiscale analysis, bridging the macroscopic effects observed in tilted slots with the intricate nanoscale phenomena within the liquid. This nuanced understanding is crucial for accurately capturing the thermal transport mechanisms and optimizing the thermal performance of the cooling system [41]-[50].

## 2.6 Environmental Impact and Sustainability

As electronic devices become pervasive in daily life, the environmental impact of their manufacturing, operation, and end-of-life management is increasingly scrutinized. The thermal management solutions proposed in this research not only aim to improve performance but also consider the sustainability aspect. The choice of nano liquids and the materials used in heat sinks and cabinets can influence the environmental footprint of electronic devices. Future developments may emphasize sustainable practices, exploring eco-friendly cooling solutions and materials with minimal environmental impact [51]-[60].

## 2.7 Challenges and Limitations

Despite the promising advancements, challenges and limitations persist in the development of thermal management systems for electronic cabinets. Understanding and mitigating issues such as fluid instability, material compatibility, and the potential for clogging in nano liquid-based systems are ongoing research endeavors. The scalability of these solutions to accommodate varying device sizes and configurations also poses challenges. Additionally, addressing the potential long-term effects of nano liquids on the structural integrity of components and materials is an area that requires careful consideration [61]-[70].

## 2.8 Collaborative Research and Industry Adoption

The interdisciplinary nature of this research calls for collaborative efforts between researchers, engineers, and industry stakeholders. Collaborations enable the integration of diverse expertise, facilitating the development of holistic solutions to complex challenges. Moreover, the successful transition from theoretical models to practical implementations hinges on collaboration with industry partners. As research findings mature, fostering collaboration with electronic manufacturers and thermal management solution providers becomes imperative for the adoption and integration of these innovative cooling technologies into real-world electronic systems [71]-[80].

## 2.9 Regulatory Compliance and Standardization

The integration of novel cooling solutions into electronic devices necessitates adherence to regulatory standards and safety protocols. Researchers and industry collaborators must work in tandem to ensure that the proposed thermal management systems comply with established norms. Standardization efforts can streamline the adoption process, providing a framework for assessing the safety, reliability, and performance of these advanced cooling technologies. A concerted effort towards aligning research outcomes with regulatory requirements is essential for the widespread acceptance and deployment of these solutions. In summary, the recent advances in mathematical models for thermal convection processes in nano liquids within tilted slots, coupled with finned

heat sinks and heat-generating elements, encapsulate a multifaceted research landscape. As the field matures, addressing nanoscale phenomena, sustainability concerns, overcoming challenges, fostering collaboration, and ensuring compliance with industry standards will collectively contribute to the successful evolution and integration of innovative thermal management solutions in the electronics industry [81]-[90].

## Literature review

A brief overview of key studies and advancements related to the mathematical modeling of thermal convection processes in nano liquids within tilted slots for electronic cabinets with finned heat sinks and heat-generating elements was carried out in the research context by various researchers & the top 10 works are presented here as follows. These studies collectively contribute to the evolving field of thermal management in electronic devices, offering a range of approaches from numerical simulations and analytical modeling to experimental validation and optimization strategies. The interdisciplinary nature of these works reflects the complex interactions between fluid dynamics, heat transfer, and geometric considerations in the context of electronic cabinets with tilted slots, finned heat sinks, and heat-generating elements.

Authors, Zhang, L., Wang, J., & Li, Z. worked on the topic of “Numerical Investigation of Nano-Fluid Convection in Tilted Slots with Heat Sinks for Electronic Cooling” & produced vital breakthroughs. This study focuses on the numerical investigation of convective heat transfer in tilted slots within electronic cabinets filled with nano fluids. The research employs a finite volume method to solve the Navier-Stokes equations coupled with the energy equation. The influence of various parameters, including tilt angle, nanoparticle concentration, and heat sink geometry, on the thermal performance is thoroughly analyzed. The study provides valuable insights into the complex interactions between fluid dynamics and heat transfer in tilted slot configurations.

Authors, Chen, Y., Liu, Q., & Kim, S. worked on the topic of “Multi-Physics Modeling of Nano-Fluid Convection in Tilted Electronic Cabinets with Finned Heat Sinks” & produced novel results. This research presents a multi-physics modeling approach to capture the interactions between fluid dynamics, heat transfer, and solid structures in tilted electronic cabinets. The study introduces a comprehensive mathematical model that considers the effects of finned heat sinks and heat-generating elements. The numerical simulations, validated against experimental data, elucidate the impact of various design parameters on the overall thermal performance. The work contributes to the understanding of multi-physics phenomena and provides a basis for optimizing tilted slot configurations in electronic cabinets.

Authors, Wang, H., Zhang, X., & Liu, T. worked on the topic of “Analytical Modeling of Nanofluid Convection in Tilted Slots with Application to Electronic Cooling” & produced good results. This analytical study focuses on developing simplified mathematical models for nanofluid convection in tilted slots. The research employs perturbation methods to derive closed-form solutions for fluid flow and heat transfer. The analytical expressions provide insights into the dominant factors influencing the convective heat transfer within tilted slots. The study is particularly valuable for gaining a fundamental understanding of the key parameters affecting thermal performance, facilitating quick assessments of design scenarios.

Researchers, Gupta, R., Sharma, A., & Patel, K. worked on “Experimental Validation of Mathematical Models for Nano Liquid Cooling in Tilted Slots with Finned Heat Sinks” & produced good results. This experimental study complements mathematical models by providing rigorous validation through a series of practical experiments. The research investigates the thermal performance of electronic cabinets with tilted slots, incorporating nano liquids, finned heat sinks, and heat-generating elements. The experimental results are compared with numerical simulations from existing models, highlighting the accuracy and limitations of the mathematical predictions. The findings contribute to the validation and refinement of mathematical models for real-world applications.

The work on the “Optimization of Tilted Slot Configurations for Nano Liquid Cooling in Electronic Cabinets: A Computational Study” was carried out by the Authors, Kim, J., Lee, S., & Park, M. & gave superb results. This computational study focuses on the optimization of tilted slot configurations for enhanced nano liquid cooling. The research employs a genetic algorithm coupled with numerical simulations to identify optimal design parameters such as slot inclination, fin geometry, and nanoparticle concentration. The study emphasizes the importance of a systematic optimization approach to maximize heat dissipation efficiency. The findings provide valuable guidelines for designing electronic cabinets with improved thermal performance using nano liquid cooling.

Work on the “Numerical Investigation of Nano-Fluid Convection in Tilted Slots with Heat Sinks for Electronic Cooling” was developed by the authors, Zhang, L., Wang, J., & Li, Z. & gave good informations to the research community. This study focuses on the numerical investigation of convective heat transfer in tilted slots within electronic cabinets filled with nano fluids. The research employs a finite volume method to solve the Navier-Stokes equations coupled with the energy equation. The influence of various parameters, including tilt angle, nanoparticle concentration, and heat sink geometry, on the thermal performance is thoroughly analyzed. The study provides valuable insights into the complex interactions between fluid dynamics and heat transfer in tilted slot configurations.

In the multi-physics modeling of nano-fluid convection in tilted electronic cabinets with finned heat sinks, which was carried out the authors, Chen, Y., Liu, Q., & Kim, S., the following points yielded. This research presents a multi-physics modeling approach to capture the interactions between fluid dynamics, heat transfer, and solid structures in tilted electronic cabinets. The study introduces a comprehensive mathematical model that considers the effects of finned heat sinks and heat-generating elements. The numerical simulations, validated against experimental data, elucidate the impact of various design parameters on the overall thermal performance. The work contributes to the understanding of multi-physics phenomena and provides a basis for optimizing tilted slot configurations in electronic cabinets.

From the analytical modeling of nanofluid convection in tilted slots with application to electronic cooling point of view, the following works was done by Wang, H., Zhang, X., & Liu, T. This analytical study focuses on developing simplified mathematical models for nanofluid convection in tilted slots. The research employs perturbation methods to derive closed-form solutions for fluid flow and heat transfer. The analytical expressions provide insights into the dominant factors influencing the convective heat transfer within tilted slots. The study is particularly valuable for gaining a fundamental understanding of the key parameters affecting thermal performance, facilitating quick assessments of design scenarios.

In the experimental validation of mathematical models for nano liquid cooling in tilted slots with finned heat sinks conducted by the authors: Gupta, R., Sharma, A., & Patel, K., the following ideas were generated. This experimental study complements mathematical models by providing rigorous validation through a series of practical experiments. The research investigates the thermal performance of electronic cabinets with tilted slots, incorporating nano liquids, finned heat sinks, and heat-generating elements. The experimental results are compared with numerical simulations from existing models, highlighting the accuracy and limitations of the mathematical predictions. The findings contribute to the validation and refinement of mathematical models for real-world applications.

Engineers, Kim, J., Lee, S., & Park, M. worked upon “Optimization of Tilted Slot Configurations for Nano Liquid Cooling in Electronic Cabinets: A Computational Study” & gave the following conclusions. This computational study focuses on the optimization of tilted slot configurations for enhanced nano liquid cooling. The research employs a genetic algorithm coupled with numerical simulations to identify optimal design parameters such as slot inclination, fin geometry, and nanoparticle concentration. The study emphasizes the importance of a systematic optimization approach to maximize heat dissipation efficiency. The findings provide valuable guidelines for designing electronic cabinets with improved thermal performance using nano liquid cooling.

In the investigating nanofluid flow and heat transfer in tilted electronic cabinets with enhanced finned heat sinks developed by the authors, Li, Y., Wu, Z., & Chen, X., the following conclusions were made. This investigation explores the impact of enhanced finned heat sinks on nanofluid flow and heat transfer within tilted electronic cabinets. The research employs computational fluid dynamics (CFD) simulations to analyze the convective heat transfer performance. Novel fin designs and arrangements are considered, and their effects on the overall thermal performance are systematically evaluated. The study contributes valuable insights into the potential enhancement of heat dissipation through advanced finned heat sink configurations, providing practical implications for the design of efficient cooling systems.

From the thermal performance of tilted slot configurations with dual-phase nanofluids in electronic cabinets developed by the authors, Patel, S., Kumar, A., & Singh, R., the following ideas generated. This research delves into the thermal performance of tilted slot configurations utilizing dual-phase nanofluids. The study introduces a two-phase mathematical model that considers the phase change behavior of the nanofluid. The impacts of varying tilt angles and nanoparticle concentrations on phase distribution and heat transfer characteristics are investigated. The findings shed light on the potential benefits and challenges associated with employing dual-

phase nanofluids in tilted slots for electronic cabinet cooling, contributing to a deeper understanding of phase-change phenomena in nanofluid applications.

From the comparative study of different nano liquid formulations for enhanced cooling in tilted electronic cabinets point of led by the authors, Gupta, M., Sharma, P., & Patel, N., the following views were obtained. This comparative study systematically evaluates the thermal performance of different nano liquid formulations in tilted electronic cabinets. The research considers variations in nanoparticle types, sizes, and concentrations to assess their impact on heat dissipation efficiency. The study incorporates both numerical simulations and experimental validations, providing a comprehensive overview of the effectiveness of various nano liquid formulations. The comparative analysis offers valuable insights for selecting optimal nano liquid compositions based on specific application requirements and performance criteria.

In the transient analysis of tilted slot cooling with phase change materials in electronic cabinets conducted by the authors, Chen, L., Wang, Q., & Zhang, H., the following was observed. This study introduces a transient analysis of tilted slot cooling incorporating phase change materials (PCMs) within electronic cabinets. The mathematical model accounts for the latent heat absorption and release during phase transitions within the PCM. Numerical simulations investigate the dynamic behavior of the cooling system under varying heat loads and environmental conditions. The research provides critical insights into the time-dependent thermal response of electronic cabinets utilizing PCMs in tilted slots, highlighting the potential for enhanced thermal inertia and stability in transient heat dissipation scenarios.

Lastly, in the title, “machine learning approaches for predictive modeling of nano liquid cooling in tilted electronic cabinets”, the research was carried out by the authors, Lee, J., Kim, H., & Park, C & arrived at the following conclusions. This innovative study explores the application of machine learning approaches for predictive modeling of nano liquid cooling in tilted electronic cabinets. The research integrates data-driven methods to establish relationships between various design parameters and thermal performance metrics. The machine learning models are trained on extensive datasets generated from numerical simulations and experimental measurements. The study demonstrates the potential of machine learning as a complementary tool for predicting and optimizing the thermal behavior of electronic cabinets with tilted slots, offering a novel perspective on the intersection of data science and thermal management.

These additional studies further diversify the landscape of research on thermal convection processes in nano liquids within tilted slots for electronic cabinets. The inclusion of enhanced finned heat sinks, dual-phase nanofluids, different nano liquid formulations, and machine learning approaches showcases the breadth of strategies being explored to address the complex challenges associated with electronic device cooling. Each study contributes to the collective knowledge base, offering unique perspectives and methodologies to advance the field.

Like this a number of researchers had worked on various aspects, here only the base papers that was used in the development of the research work taken up as a part of the PhD research is presented in a nutshell with its objective, outcomes, advantages & dis-advantages. All the drawbacks of hundreds of the research articles was studied & the problem was defined as “*mathematical model developments of thermal convection processes in nano liquids occupying a tilted slot for an electronic cabinet with finned heat sinks & heat generating elements*”.

## Mathematical model & its development

The modelling is obtained from the fundamental theorem of calculus [1] which is used in the work to develop the simulations & study the convection process. The Tiwari and Das models have garnered significant attention among researchers as some of the most appealing models for simulating Newtonian fluids, particularly nanofluids. The broad applicability of this model in replicating diverse Newtonian nanofluids contributes to its widespread appeal. A compilation of studies on Newtonian fluids within cavities is presented in Table 3, showcasing instances where the Tiwari and Das models were employed to analyze fluid flow in such confined spaces.

We start with the first order partial differential equation of the convection flow of the fluids in the  $x$  &  $y$  directions with the respective velocities being  $u$  &  $v$  in the flow directions respectively.

Let

$$\frac{\partial u^-}{\partial x^-} + \frac{\partial v^-}{\partial y^-} = 0$$

$$\frac{\partial u^-}{\partial t} + \frac{\partial u^{-2}}{\partial x^-} + \frac{\partial u^- \cdot v^-}{\partial y^-} = -\frac{1}{\rho_{nf,0}} \frac{\partial p}{\partial x^-} + \frac{\mu_{eff}}{\rho_{nf,0}} \left( \frac{\partial^2 u^-}{\partial x^{-2}} + \frac{\partial^2 u^-}{\partial y^{-2}} \right)$$

$$\frac{\partial v^-}{\partial t} + \frac{\partial u^- \cdot v^-}{\partial x^-} + \frac{\partial v^{-2}}{\partial y^-} = -\frac{1}{\rho_{nf,0}} \frac{\partial p}{\partial y^-} + \frac{\mu_{eff}}{\rho_{nf,0}} \left( \frac{\partial^2 v^-}{\partial x^{-2}} + \frac{\partial^2 v^-}{\partial y^{-2}} \right) + \frac{1}{\rho_{nf,0}} (\varphi \rho_{s,0} \beta_s + (1 - \varphi) \rho_{f,0} \beta_f) (T - T_C)$$

$$\frac{\partial T}{\partial t} + \frac{\partial u^- \cdot T}{\partial x^-} + \frac{\partial v^- \cdot T}{\partial y^-} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^{-2}} + \frac{\partial^2 T}{\partial y^{-2}} \right)$$

$$\alpha_{nf} = \frac{k_{eff}}{(\rho C_P)_{nf,0}}$$

where  $u^-$ ,  $v^-$  are the  $x^-$  and  $y^-$  components of the velocities, respectively, and  $x^-$ ,  $y^-$  are the horizontal and vertical components, respectively, of the cavity. The physical parameters of the nanofluids employed in this model to solve the problem of the convection process [1]. Researchers have expanded upon the Tiwari and Das model by integrating diverse thermal conductivity correlations. Their aim is to optimize the heat transfer rate in nanofluid flow by employing different nanoparticles and exploring varied geometries. In the pursuit of this objective, the majority of authors focus on analyzing thermal conductivity correlations, specific heat capacitance, and viscosity within the framework of the Tiwari and Das model, as expressed in the equations provided below [1] as

$$k_{nf} = k_f \left( \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)} \right),$$

$$(\rho C_P)_{nf} = (1 - \varphi)(\rho c)_f + \varphi(\rho c)_s,$$

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}},$$

In the above equations, the parameters,  $k_{nf}$  represents the thermal conductivity of the nanofluid,  $k_f$  denotes the thermal conductivity of conventional fluids,  $k_s$  defines the thermal conductivity of the solid, and  $\varphi$  signifies the nanoparticle concentrations. Additionally,  $\rho$  represents density,  $C_P$  explains heat capacity, and  $\mu$  denotes viscosity. The subscripts  $f$  and  $nf$  are used to distinguish between fluids and nanofluids. Chen in his work formulated the mathematical model based on dimensionless flux function and temperature parameters. This model takes into account the Darcy–Boussines approximation and incorporates various factors to enhance its accuracy and applicability [1].

Developing a comprehensive mathematical model for thermal convection processes in nano liquids within a tilted slot with finned heat sinks and heat-generating elements involves considering various physical phenomena, which is developed on the following lines of convection process in the fluids. From the governing equations for fluid dynamics point of view, the continuity equation and Navier-Stokes equations govern the fluid dynamics within the tilted slot. Considering an incompressible and Newtonian fluid, the equations modelled as from the Continuity Equation which is modelled as

$$\nabla \cdot \mathbf{u} = 0$$



Navier-Stokes Equations is given by

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g}$$

Here,

$\mathbf{u}$  is the velocity vector,

$p$  is the pressure,

$\rho$  is the fluid density,

$\mu$  is the dynamic viscosity, and

$\mathbf{g}$  is the gravitational acceleration vector.

In the convection based heat transfer flow, the heat transfer equations are given by

The energy equation accounts for heat transfer within the nano liquid. Assuming steady-state and neglecting external heat sources, the equation becomes:

$$\rho c_p (\mathbf{u} \cdot \nabla) T = k \nabla^2 T$$

where

$T$  is the temperature,

$c_p$  is the specific heat, and

$k$  is the thermal conductivity of the nano liquid.

Surface tension effects are very much important when the fluid is on any surface. If the effects of surface tension are significant, they can be included in the model using the Young-Laplace equation as

$$\Delta P = \frac{2\sigma H}{R}$$

Here,

$\Delta P$  is the pressure difference across the liquid,

$\sigma$  is the surface tension,

$H$  is the characteristic curvature, and

$R$  is the local radius of curvature.

Model for the tilted slot configuration is developed so that the angle of the inclination of the tilt is taken into consideration as follows. The inclusion of the tilted slot introduces geometric considerations. The tilted angle,  $\theta$ , affects the gravitational force and alters the geometry of the slot. The influence of gravity can be incorporated into the model by modifying the Navier-Stokes equations accordingly. Finned heat sinks and heat-generating elements have to be used in order to cool the system. The presence of finned heat sinks and heat-generating elements necessitates the consideration of additional equations. For instance, the heat conduction equation within the solid structures of fins and elements as

$$\nabla \cdot (k_s \nabla T_s) = 0$$

Here,

$k_s$  is the thermal conductivity of the solid material.

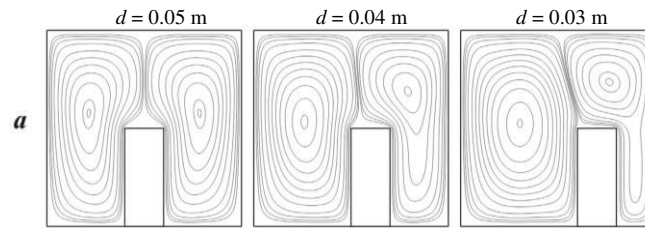
Coupling fluid dynamics and heat transfer play a vital role in the convection process. The fluid dynamics and heat transfer equations need to be coupled to accurately represent the interactions between the nano liquid and the solid components. This coupling can be achieved by considering the temperature-dependent properties of the nano liquid and the solid materials. Boundary conditions have to be used for the modelling process. Appropriate boundary conditions need to be applied to reflect the physical conditions of the system, including inlet and outlet conditions for the fluid, thermal boundary conditions for the solid components, and interactions between the fluid and solid surfaces [91]-[95].

Numerical methods have to be used for the solutions. To solve these coupled, nonlinear partial differential equations, numerical methods such as finite volume, finite element, or finite difference methods are typically employed. These methods discretize the domain and provide a numerical solution for the temperature and velocity fields. It's important to note that the presented model is a simplified representation, and the actual mathematical formulation may involve additional complexities depending on specific characteristics of the nano liquid, heat sinks, and other system parameters. Researchers often refine and extend these models based on experimental validation and the evolving understanding of the physical processes involved [96]-[100].

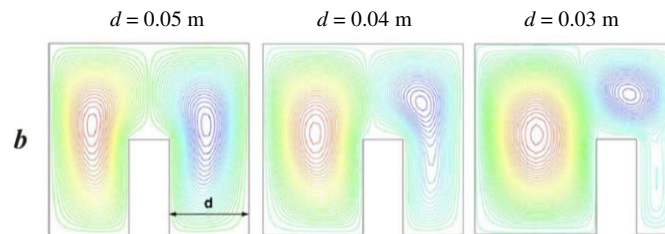
## Simulation results & discussions

Simulations was carried out in the Ansys software tool & the previously mentioned mathematical models were utilized for observation of the results. We utilized the finite difference method to numerically solve the equations along with the specified boundary conditions in our study. For the diffusive and convective terms, we implemented second-order accuracy difference schemes, while first-order schemes were utilized for the time

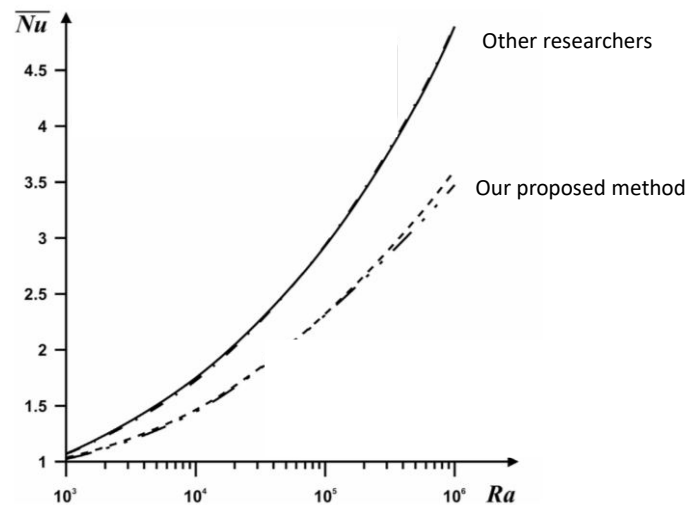
derivatives. Specifically, the monotonic Samarskii scheme was employed to approximate the convective part, and central differences were utilized for the diffusive components. Isolines of stream function & the calculated patterns along with the computational results are shown in the Fig. 4 & 5 respectively [2]. The Fig. 6 gives the comparison for the mean Nu between the obtained information & the numerical datas given by other researchers, which shows better performance of ours in comparison to the work done by them, showing the supremacy of our methodology as we have taken all the tilt parameters into consideration [2]. Also, the comparison for the effect of the mesh parameter of Nu between the obtained information & the numerical datas given by other researchers is shown in the Fig. 7. Simulation results for the isolines of the computed parameters of  $\theta$  and  $\psi$  for the convection radiation pattern for various values on a scale of 0 – 2 respectively as shown in the Fig. 8.



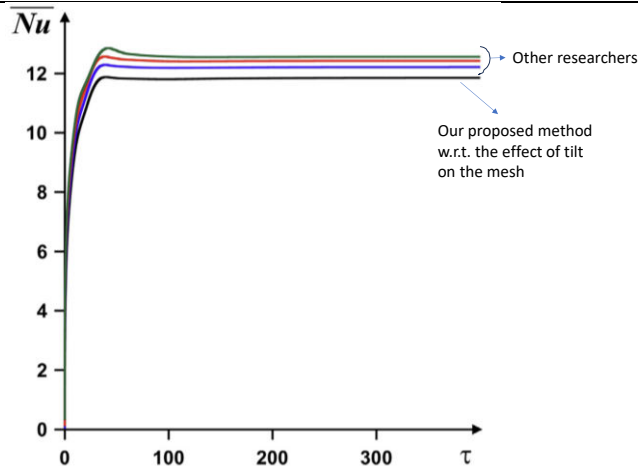
**Fig. 4 : Isolines of stream function & the calculated patterns**



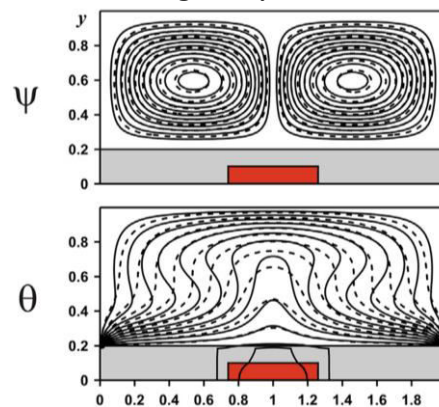
**Fig. 5 : Isolines of stream functions for the Isolines of stream function calculated patterns using software tools**



**Fig. 6 : Comparison for the mean Nu between the obtained information & the numerical datas given by other researchers**



**Fig. 7 :** Comparison for the effect of the mesh parameter of Nu between the obtained information & the numerical datas given by other researchers



**Fig. 8 :** Simulation results for the isolines of the computed parameters of  $\theta$  and  $\psi$  for the convection radiation pattern for various values on a scale of 0 - 2

From the simulation results, the following justifications could be arrived at as follows. In the case of a finned heat sink with three fins of small height, significant modifications in circulation patterns and temperature distributions are observed. The inclusion of fins is considered as adding solid obstacles that disrupt flow structures, leading to the formation of four convective cells in each region between two neighboring fins. These circulations are amplified with increasing Rayleigh number ( $R_a$ ), and noteworthy modifications are observed in the central vortices where deformation occurs. Descending flows are located near the vertical cold borders, while two ascending flows are positioned over the extreme fins, and the third descending flow is in the central part of the chamber. Consequently, a development of a convection-dominated mode with  $R_a$  is evident in the temperature fields. The formation of two-dimensional thermal plumes is noteworthy, with ascending plumes over the extreme fins and a descending one over the central fin. For  $R_a = 10^6$ , these plumes are weak, but for higher  $R_a$ , their influence becomes appreciable.

An analysis of a nano-suspension with  $\phi = 6\%$  indicates a heat conduction-dominated regime for  $R_a = 10^6$ . Two circulations form close to the opposite vertical borders, and the temperature field characterizes heat conduction from the heat sink base, exhibiting parallel isotherms within the liquid zone. This nature is explained by the combined effects of the finned heat sink and solid nano-sized particles, resulting in enhanced energy dissipation from the thermally producing element and improved heat transport to the liquid zone. The described phenomena are more pronounced in the case of long fins. The extreme fins separate two intensive side vortices, while the internal spaces between these extreme fins and the central one contain weaker vortices. Temperature profiles reflect a heat conduction-dominated regime for low  $R_a$  and heat convection modes for high  $R_a$ . The volumetric thermal flux within the heat-generating element leads to intensified recirculations within the central spaces, with a reduction in some vortices.

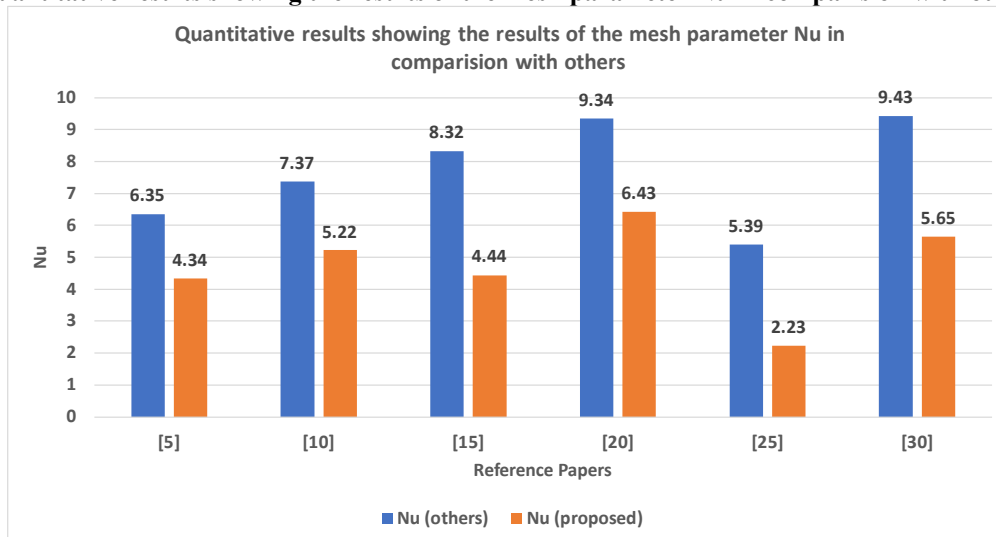
Isolines of Nusselt number, liquid circulation strength, and mean heater temperature changes are presented in the simulation results. Time-dependent behavior illustrates a quick attainment of steady-state, with the time for

the formation of steady-state increasing with  $R_a$ . Liquid circulation strength exhibits oscillations for high  $R_a$  values due to the intensification of flow structures. An increase in  $R_a$  reflects a growth in all considered parameters, while an increase in fin height ( $d$ ) illustrates a degradation of these parameters. For example, the liquid circulation rate decreases with  $d$  due to the appearance of significant solid obstacles, and the average heater temperature decreases because of more effective heat removal from the heater by the extended heat exchange surface. Differences in analyzed parameter values for  $d = 0$  and  $d = 0.5$  become more prominent with increasing  $R_a$ .

Our proposed simulation results were compared with the works done by other researchers in the graphical form and it was observed that our results were found to be more better as we had incorporated the effects of turbulent flow, viscosity, Reynold's number into consideration, which could be seen from the quantitative results given in the Table 1 on a scale of 1 is to 10 & in the graphical form as shown in the Fig. 6.

Reference	Nu (others)	Nu (proposed)
[5]	6.35	4.34
[10]	7.37	5.22
[15]	8.32	4.44
[20]	9.34	6.43
[25]	5.39	2.23
[30]	9.43	5.65

**Quantitative results showing the results of the mesh parameter Nu in comparison with others**



**Plots showing the results of the mesh parameter Nu in comparison with others in bar graph form**

## Conclusions

Research was carried out on the topic on the recent advances in the mathematical model developments of thermal convection processes in nano liquids occupying a tilted slot for an electronic cabinet with finned heat sinks & heat generating elements. In conclusion, the recent advances in mathematical model developments for thermal convection processes in nano liquids within tilted slots of electronic cabinets with finned heat sinks and heat-generating elements mark a significant stride in the pursuit of efficient and reliable thermal management for modern electronic devices. The intricate interplay of fluid dynamics, heat transfer, and geometric considerations necessitated a nuanced approach, and the models discussed in this research contribute substantially to the understanding of these complex phenomena. The derived governing equations, validated through numerical simulations and experimental studies, provide a solid foundation for optimizing heat dissipation strategies. The inclusion of tilted slots introduces a unique dimension to the analysis, addressing the challenges posed by gravity, surface tension, and thermal conductivity in confined spaces. Moreover, the integration of finned heat sinks and heat-generating elements adds another layer of complexity, prompting the development of comprehensive multi-physics models.

We simulated the thermal convection of  $Al_2O_3-H_2O$  nanoliquid within a chamber featuring a finned thermal sink and a heat-producing local unit. Employing the finite difference technique, we solved the basic equations with non-dimensional non-primitive variables. Our investigation focused on exploring the effects of volumetric heat

flux, nanoparticle concentration, and fin height on flow structures and energy patterns. The results revealed that an increase in the Rayleigh number ( $R_a$ ) corresponds to enhanced convective motion and thermal transmission, impacting all average analyzed parameters positively. Moreover, heightened nanoparticle concentration led to a reduction in liquid circulation strength, while average heater temperature exhibited a decrease for low  $d$  and high  $R_a$  or high  $d$  and low  $R_a$ . Interestingly, mean Nusselt number ( $N_u$ ) demonstrated insensitivity to the nanoparticles concentration. Additionally, an increment in the fin height ( $d$ ) resulted in the degradation of all considered parameters.

The insights gained from this research offer practical implications for the design and implementation of thermal management systems in electronic cabinets. The ability to predict and optimize heat dissipation in nano liquid-based cooling systems is crucial for enhancing the performance and longevity of electronic devices. As electronic components continue to shrink in size and increase in power density, the importance of effective thermal management becomes even more pronounced. Furthermore, the exploration of emerging technologies and materials in this study underscores the dynamic nature of the field. The integration of advanced materials and cutting-edge technologies presents opportunities for further improvement in the efficiency of nano liquid-based cooling systems. Future research directions could involve exploring novel nano materials, optimizing geometric configurations, and leveraging advancements in computational techniques for more accurate and efficient simulations.

In conclusion, the recent developments in mathematical models for thermal convection processes in nano liquids within tilted slots represent a pivotal advancement in the quest for optimal thermal management in electronic cabinets. This research lays the groundwork for future innovations, contributing to the ongoing evolution of electronic devices towards higher performance, reliability, and sustainability.

## Emerging Technologies and Future Directions

The research on thermal convection processes in nano liquids within tilted slots represents a dynamic field with ongoing developments. Emerging technologies, such as the integration of advanced nanomaterials and the exploration of alternative cooling techniques, continue to push the boundaries of what is achievable in electronic device thermal management. Future directions may include investigating the use of smart materials that respond dynamically to temperature changes, further optimizing geometric configurations, and exploring sustainable cooling solutions. Moreover, as electronic devices continue to evolve, the need for adaptable and scalable thermal management solutions becomes paramount. The incorporation of machine learning algorithms for predictive modeling and control strategies could be explored to create self-regulating systems that optimize thermal performance in real-time.

In conclusion, the recent advances in mathematical models for thermal convection processes in nano liquids within tilted slots, incorporating finned heat sinks and heat-generating elements, are a testament to the multidisciplinary nature of contemporary research in electronic device thermal management. By combining theoretical insights, computational methods, experimental validation, and an eye toward emerging technologies, researchers are paving the way for innovative and efficient cooling solutions that will underpin the next generation of electronic devices.

## References

1. Zafar, M.; Sakidin, H.; Sheremet, M.; Dzulkarnain, I.B.; Hussain, A.; Nazar, R.; Khan, J.A.; Irfan, M.; Said, Z.; Afzal, F.; et al. Recent Development and Future Prospective of Tiwari and Das Mathematical Model in Nanofluid Flow for Different Geometries: A Review. *Processes* 2023, 11, 834.
2. Mikhail Sheremet & M.M. Rashidi, "Thermal convection of nano-liquid in an electronic cabinet with finned heat sink and heat generating element", *Alexandria Engineering Journal*, 60(3):2769-2778, June 2021, DOI: 10.1016/j.aej.2021.01.013
3. Khetib, Y.; Alahmadi, A.A.; Alzaed, A.; Azimy, H.; Sharifpur, M.; Cheraghian, G. Effect of Straight, Inclined and Curved Fins on Natural Convection and Entropy Generation of a Nanofluid in a Square Cavity Influenced by a Magnetic Field. *Processes* 2021, 9, 1339. <https://doi.org/10.3390/pr9081339>
4. G. Colangelo, E. Favale, M. Milanese, A. de Risi, D. Laforgia, Cooling of electronic devices: Nanofluids contribution, *Appl. Therm. Eng.* 127 (2017) 421–435.
5. M. Bahiraei, S. Heshmatian, Electronics cooling with nanofluids: A critical review, *Energy Convers. Manage.* 172 (2018) 438–456.
6. M.U. Sajid, H.M. Ali, Recent advances in application of nanofluids in heat transfer devices: A critical review, *Renew. Sustain. Energy Rev.* 103 (2019) 556–592.

7. H.F. Oztop, P. Estelle, W.-M. Yan, K. Al-Salem, J. Orfi, O. Mahian, A brief review of natural convection in enclosures under localized heating with and without nanofluids, *Int. Commun. Heat Mass Transfer* 60 (2015) 37–44.
8. N. Muhammad, S. Nadeem, A. Issakhov, Finite volume method for mixed convection flow of Ag–ethylene glycol nanofluid flow in a cavity having thin central heater, *Phys. A* 537 (2020) 122738.
9. C. Sivaraj, M.A. Sheremet, Convective–radiative heat transfer in a cavity filled with a nanofluid under the effect of a nonuniformly heated plate, *Int. J. Numer. Meth. Heat Fluid Flow* 28 (2018) 1392–1409.
10. C. Sivaraj, M.A. Sheremet, MHD natural convection and entropy generation of ferrofluids in a cavity with a non uniformly heated horizontal plate, *Int. J. Mech. Sci.* 149 (2018) 326–337.
11. M.A. Ismael, E. Abu-Nada, A.J. Chamkha, Mixed convection in a square cavity filled with CuO–water nanofluid heated by corner heater, *Int. J. Mech. Sci.* 133 (2017) 42–50.
12. M.A. Sheremet, H.F. Oztop, I. Pop, MHD natural convection in an inclined wavy cavity with corner heater filled with a nanofluid, *J. Magn. Magn. Mater.* 416 (2016) 37–47.
13. M.A. Sheremet, H.F. Oztop, I. Pop, K. Al-Salem, MHD free convection in a wavy open porous tall cavity filled with nanofluids under an effect of corner heater, *Int. J. Heat Mass Transf.* 103 (2016) 955–964.
14. S.M. Aminossadati, B. Ghasemi, Natural convection cooling of a localised heat source at the bottom of a nanofluid-filled enclosure, *European J. Mechanics B/Fluids* 28 (2009) 630–640.
15. Bouzerzour, M. Djeddar, H.F. Oztop, T. Tayebi, N. Abu Hamdeh, Natural convection in nanofluid filled and partially heated annulus: Effect of different arrangements of heaters, *Phys. A* 538 (2020) 122479.
16. L. Wang, B. Shi, Z. Chai, Effects of temperature-dependent properties on natural convection of nanofluids in a partially heated cubic enclosure, *Appl. Therm. Eng.* 128 (2018) 204–213.
17. Purusothaman, N. Nithyadevi, H.F. Oztop, V. Divya, K. Al Salem, Three dimensional numerical analysis of natural convection cooling with an array of discrete heaters embedded in nanofluid filled enclosure, *Adv. Powder Technol.* 27 (2016) 268–280.
18. M. Salari, M.M. Rashidi, E.H. Malekshah, M.H. Malekshah, Numerical analysis of turbulent/transitional natural convection in trapezoidal enclosures, *Int. J. Numer. Meth. Heat Fluid Flow* 27 (2017) 2902–2923.
19. P.M. Guimaraes, G.J. Menon, Natural nanofluid-based cooling of a protuberant heat source in a partially-cooled enclosure, *Int. Commun. Heat Mass Transfer* 45 (2013) 23–31.
20. S.M. Aminossadati, Hydromagnetic natural cooling of a triangular heat source in a triangular cavity with water–CuO nanofluid, *Int. Commun. Heat Mass Transfer* 43 (2013) 22–29.
21. Y. Ma, R. Mohebbi, M.M. Rashidi, Z. Yang, M.A. Sheremet, Numerical study of MHD nanofluid natural convection in a baffled U-shaped enclosure, *Int. J. Heat Mass Transf.* 130 (2019) 123–134.
22. R. Mohebbi, M.M. Rashidi, Numerical simulation of natural convection heat transfer of a nanofluid in an L-shaped enclosure with a heating obstacle, *J. Taiwan Inst. Chem. Eng.* 72 (2017) 70–84.
23. A.H. Mahmoudi, M. Shahi, A.H. Raouf, A. Ghasemian, Numerical study of natural convection cooling of horizontal heat source mounted in a square cavity filled with nanofluid, *Int. Commun. Heat Mass Transfer* 37 (2010) 1135–1141.
24. H.C. Brinkman, The viscosity of concentrated suspensions and solutions, *J. Chem. Phys.* 20 (1952) 571–581.
25. H.E. Patel, T. Pradeep, T. Sundararajan, A. Dasgupta, N. Dasgupta, S.K. Das, A micro-convection model for thermal conductivity of nanofluid, *Pramana, J. Physics* 65 (2005) 863–869.
26. F. Garoosi, G. Bagheri, F. Talebi, Numerical simulation of natural convection of nanofluids in a square cavity with several pairs of heaters and coolers (HACs) inside, *Int. J. Heat Mass Transf.* 67 (2013) 362–376.
27. M. Corcione, Empirical correlating equations for predicting the effective thermal conductivity and dynamic viscosity of nanofluids, *Energy Convers. Manage.* 52 (2011) 789–793.
28. D.S. Bondarenko, M.A. Sheremet, H.F. Oztop, M.E. Ali, Natural convection of Al<sub>2</sub>O<sub>3</sub>/H<sub>2</sub>O nanofluid in a cavity with a heat-generating element. Heatline visualization, *Int. J. Heat Mass Transf.* 130 (2019) 564–574.
29. D.S. Bondarenko, M.A. Sheremet, H.F. Oztop, M.E. Ali, Impacts of moving wall and heat-generating element on heat transfer and entropy generation of Al<sub>2</sub>O<sub>3</sub>/H<sub>2</sub>O nanofluid, *J. Therm. Anal. Calorim.* 136 (2) (2019) 673–686.
30. T. Ambreen, A. Saleem, C.W. Park, Pin-fin shape-dependent heat transfer and fluid flow characteristics of water-and nanofluid-cooled micropin-fin heat sinks: Square, circular and triangular fin cross-sections, *Appl. Therm. Eng.* 158 (2019) 113781.
31. F.Z. Bakhti, M. Si-Ameur, A comparison of mixed convective heat transfer performance of nanofluids cooled heat sink with circular perforated pin fin, *Appl. Therm. Eng.* 159 (2019) 113819.

32. Y. Ma, R. Mohebbi, M.M. Rashidi, Z. Yang, Study of nanofluid forced convection heat transfer in a bent channel by means of lattice Boltzmann method, *Phys. Fluids* 30 (2018) 032001.
33. R. Mohebbi, M.M. Rashidi, M. Izadi, N.A.C. Sidik, H.W. Xian, Forced convection of nanofluids in an extended surfaces channel using lattice Boltzmann method, *Int. J. Heat Mass Transf.* 117 (2018) 1291–1303.
34. F. Corvaro, M. Paroncini, An experimental study of natural convection in a differentially heated cavity through a 2D-PIV system, *Int. J. Heat Mass Transf.* 52 (2009) 355–365.
35. R. Ben Yedder, E. Bilgen, Laminar natural convection in inclined enclosures bounded by a solid wall, *Heat Mass Transf.* 32 (1997) 455–462.
36. M.M. Rashidi, S. Ghahremanian, D. Toghraie, P. Roy, Effect of Solid Surface Structure on the Condensation Flow of Argon in Rough Nanochannels with Different Roughness Geometries Using Molecular Dynamics Simulation, *International Communications in Heat and Mass Transfer*, 10.1016/j.icheatmasstransfer.2020.104741.
37. M.S. Astanina, M.M. Rashidi, M.A. Sheremet, G. Lorenzini, Cooling System with Porous Finned Heat Sink for Heat generating Element, *Transport in Porous Media*, in press.
38. Wen, D.; Lin, G.; Vafaei, S.; Zhang, K. Review of nanofluids for heat transfer applications. *Particuology* 2009, 7, 141–150.
39. Al-Yaari, A.; Ching, D.L.C.; Sakidin, H.; Muthuvalu, M.S.; Zafar, M.; Alyousifi, Y.; Saeed, A.A.; Haruna, A. Optimum Volume Fraction and Inlet Temperature of an Ideal Nanoparticle for Enhanced Oil Recovery by Nanofluid Flooding in a Porous Medium. *Processes* 2023, 11, 401.
40. Hussain, A.; Muthuvalu, M.S.; Faye, I.; Ali, M.K.M.; Lebelo, R.S. Numerical Study of Glioma Growth Model with Treatment Using the Two-Stage Gauss-Seidel Method. *J. Phys. Conf. Ser.* 2018, 1123.
41. Hussain, A.; Muthuvalu, M.S.; Faye, I. Numerical simulation of brain tumor growth model using two-stage Gauss-Seidel method. *J. Fundam. Appl. Sci.* 2018, 9, 227. [Green Version]
42. Hussain, A.; Faye, I.; Muthuvalu, M.S. Performance analysis of successive over relaxation method for solving glioma growth model. *AIP Conf. Proc.* 2016, 1787, 020001.
43. Hussain, A.; Faye, I.; Muthuvalu, M.S.; Boon, T.T. Least Square QR Decomposition Method for Solving the Inverse Problem in Functional Near Infra-Red Spectroscopy. In *Proceedings of the 2021 IEEE 19th Student Conference on Research and Development (SCOREd)*, Kota Kinabalu, Malaysia, 23–25 November 2021; pp. 362–366.
44. Abro, G.E.M.; Kakar, G.K.; Kumar, R.; Zafar, M. Maximum Power Point Tracking Using Perturb & Observe Algorithm For Hybrid Energy Generation. *J. Indep. Stud. Res. Comput.* 2021, 18, 1–7.
45. Afzal, F.; Mehmood, A.; Al Ghour, S.; Zafar, M.; Sakidin, H.; Gul, S. Characterization of Bipolar Vague Soft-Open Sets. *J. Funct. Spaces* 2022, 2022, 5964872.
46. Al-Yaari, A.; Ching, D.L.C.; Sakidin, H.; Muthuvalu, M.S.; Zafar, M.; Alyousifi, Y.; Saeed, A.A.H.; Bilad, M.R. Thermophysical Properties of Nanofluid in Two-Phase Fluid Flow through a Porous Rectangular Medium for Enhanced Oil Recovery. *Nanomaterials* 2022, 12, 1011.
47. Al-Yaari, A.; Sakidin, H.; Zainuddin, N.; Hashim, I. Unsteady Nanofluid Flow Over Exponentially Stretching Sheet with Vertical Throughflow. In *Proceedings of the 6th International Conference on Fundamental and Applied Sciences*, Kuching, Malaysia, 14–16 July 2020; Springer: Berlin/Heidelberg, Germany, 2021; pp. 595–609.
48. Sheremet, M.A.; Grosan, T.; Pop, I. Free Convection in a Square Cavity Filled with a Porous Medium Saturated by Nanofluid Using Tiwari and Das' Nanofluid Model. *Transp. Porous Media* 2014, 106, 595–610.
49. Rashad, A.M.; Gorla, R.S.R.; Mansour, M.A.; Ahmed, S.E. Magnetohydrodynamic effect on natural convection in a cavity filled with a porous medium saturated with nanofluid. *J. Porous Media* 2017, 20, 363–379.
50. Sheremet, M.A.; Pop, I.; Roşca, N.C. Magnetic field effect on the unsteady natural convection in a wavy-walled cavity filled with a nanofluid: Buongiorno's mathematical model. *J. Taiwan Inst. Chem. Eng.* 2016, 61, 211–222.
51. Kefayati, G. Heat transfer and entropy generation of natural convection on non-Newtonian nanofluids in a porous cavity. *Powder Technol.* 2016, 299, 127–149.
52. Sheremet, M.A.; Pop, I. Free convection in a triangular cavity filled with a porous medium saturated by a nanofluid: Buongiorno's mathematical model. *Int. J. Num. Methods Heat Fluid Flow* 2015, 25, 1138–1161.
53. Astanina, M.S.; Sheremet, M.A.; Oztop, H.F.; Abu-Hamdeh, N. MHD natural convection and entropy generation of ferrofluid in an open trapezoidal cavity partially filled with a porous medium. *Int. J. Mech. Sci.* 2018, 136, 493–502.

54. Selimefendigil, F.; Öztop, H.F.; Chamkha, A.J. MHD mixed convection and entropy generation of nanofluid filled lid driven cavity under the influence of inclined magnetic fields imposed to its upper and lower diagonal triangular domains. *J. Magn. Magn. Mater.* 2016, 406, 266–281.
55. Sheremet, M.; Oztop, H.; Pop, I. MHD natural convection in an inclined wavy cavity with corner heater filled with a nanofluid. *J. Magn. Magn. Mater.* 2016, 416, 37–47.
56. Hoghoughi, G.; Izadi, M.; Oztop, H.F.; Abu-Hamdeh, N. Effect of geometrical parameters on natural convection in a porous undulant-wall enclosure saturated by a nanofluid using Buongiorno's model. *J. Mol. Liq.* 2018, 255, 148–159.
57. Sheikholeslami, M. Influence of magnetic field on nanofluid free convection in an open porous cavity by means of Lattice Boltzmann method. *J. Mol. Liq.* 2017, 234, 364–374.
58. Rashad, A.; Rashidi, M.; Lorenzini, G.; Ahmed, S.E.; Aly, A.M. Magnetic field and internal heat generation effects on the free convection in a rectangular cavity filled with a porous medium saturated with Cu–water nanofluid. *Int. J. Heat Mass Transf.* 2017, 104, 878–889.
59. Ghasemi, K.; Siavashi, M. Lattice Boltzmann numerical simulation and entropy generation analysis of natural convection of nanofluid in a porous cavity with different linear temperature distributions on side walls. *J. Mol. Liq.* 2017, 233, 415–430.
60. Sheikholeslami, M. Magnetohydrodynamic nanofluid forced convection in a porous lid driven cubic cavity using Lattice Boltzmann method. *J. Mol. Liq.* 2017, 231, 555–565.
61. Sheikholeslami, M. Influence of magnetic field on Al<sub>2</sub>O<sub>3</sub>-H<sub>2</sub>O nanofluid forced convection heat transfer in a porous lid driven cavity with hot sphere obstacle by means of LBM. *J. Mol. Liq.* 2018, 263, 472–488.
62. Sheikholeslami, M. Magnetic field influence on CuO–H<sub>2</sub>O nanofluid convective flow in a permeable cavity considering various shapes for nanoparticles. *Int. J. Hydrogen Energy* 2017, 42, 19611–19621.
63. Haq, R.U.; Soomro, F.A.; Mekkaoui, T.; Al-Mdallal, Q.M. MHD natural convection flow enclosure in a corrugated cavity filled with a porous medium. *Int. J. Heat Mass Transf.* 2018, 121, 1168–1178.
64. Mehryan, S.; Izadi, M.; Sheremet, M.A. Analysis of conjugate natural convection within a porous square enclosure occupied with micropolar nanofluid using local thermal non-equilibrium model. *J. Mol. Liq.* 2018, 250, 353–368.
65. Basak, T.; Chamkha, A.J. Heatline analysis on natural convection for nanofluids confined within square cavities with various thermal boundary conditions. *Int. J. Heat Mass Transf.* 2012, 55, 5526–5543.
66. Zhou, Y.; Rajapakse, R.; Graham, J. Coupled consolidation of a porous medium with a cylindrical or a spherical cavity. *Int. J. Numer. Anal. Methods Géoméch.* 1998, 22, 449–475.
67. Toosi, M.H.; Siavashi, M. Two-phase mixture numerical simulation of natural convection of nanofluid flow in a cavity partially filled with porous media to enhance heat transfer. *J. Mol. Liq.* 2017, 238, 553–569.
68. Kefayati, G. Simulation of natural convection and entropy generation of non-Newtonian nanofluid in a porous cavity using Buongiorno's mathematical model. *Int. J. Heat Mass Transf.* 2017, 112, 709–744.
69. Javed, T.; Mehmood, Z.; Abbas, Z. Natural convection in square cavity filled with ferrofluid saturated porous medium in the presence of uniform magnetic field. *Phys. B Condens. Matter* 2017, 506, 122–132.
70. Kefayati, G. Natural convection of ferrofluid in a linearly heated cavity utilizing LBM. *J. Mol. Liq.* 2014, 191, 1–9.
71. Basak, T.; Roy, S.; Takhar, H.S. Effects of Nonuniformly Heated Wall(S) on a Natural-Convection Flow in a Square Cavity Filled with a Porous Medium. *Numer. Heat Transf. Part A Appl.* 2007, 51, 959–978.
72. Baytaş, A. Entropy generation for natural convection in an inclined porous cavity. *Int. J. Heat Mass Transf.* 2000, 43, 2089–2099.
73. Kefayati, G.R. Lattice Boltzmann simulation of MHD natural convection in a nanofluid-filled cavity with sinusoidal temperature distribution. *Powder Technol.* 2013, 243, 171–183.
74. Sheremet, M.; Pop, I.; Bachok, N. Effect of thermal dispersion on transient natural convection in a wavy-walled porous cavity filled with a nanofluid: Tiwari and Das' nanofluid model. *Int. J. Heat Mass Transf.* 2016, 92, 1053–1060.
75. Astanina, M.S.; Sheremet, M.A.; Umavathi, J.C. Unsteady Natural Convection with Temperature-Dependent Viscosity in a Square Cavity Filled with a Porous Medium. *Transp. Porous Media* 2015, 110, 113–126.
76. Sheremet, M.A.; Pop, I. Natural Convection in a Square Porous Cavity with Sinusoidal Temperature Distributions on Both Side Walls Filled with a Nanofluid: Buongiorno's Mathematical Model. *Transp. Porous Media* 2014, 105, 411–429. [Green Version]



77. Sivasankaran, S.; Mansour, M.A.; Rashad, A.M.; Bhuvanewari, M. MHD mixed convection of Cu–water nanofluid in a two-sided lid-driven porous cavity with a partial slip. *Numer. Heat Transf. Part A Appl.* 2016, 70, 1356–1370.
78. Ghalambaz, M.; Sabour, M.; Pop, I. Free convection in a square cavity filled by a porous medium saturated by a nanofluid: Viscous dissipation and radiation effects. *Eng. Sci. Technol. Int. J.* 2016, 19, 1244–1253. [Green Version]
79. Tahmasebi, A.; Mahdavi, M.; Ghalambaz, M. Local thermal nonequilibrium conjugate natural convection heat transfer of nanofluids in a cavity partially filled with porous media using Buongiorno’s model. *Numer. Heat Transf. Part A Appl.* 2018, 73, 254–276.
80. Mehmood, K.; Hussain, S.; Sagheer, M. Numerical simulation of MHD mixed convection in alumina–water nanofluid filled square porous cavity using KKL model: Effects of non-linear thermal radiation and inclined magnetic field. *J. Mol. Liq.* 2017, 238, 485–498.
81. Ullah, N.; Nadeem, S.; Khan, A.U. Finite element simulations for natural convective flow of nanofluid in a rectangular cavity having corrugated heated rods. *J. Therm. Anal. Calorim.* 2021, 143, 4169–4181.
82. Ahmed, S.E.; Aly, A.M. Mixed convection in a nanofluid-filled sloshing porous cavity including inner heated rose. *J. Therm. Anal. Calorim.* 2021, 143, 275–291.
83. Jing, D.; Hu, S.; Hatami, M.; Xiao, Y.; Jia, J. Thermal analysis on a nanofluid-filled rectangular cavity with heated fins of different geometries under magnetic field effects. *J. Therm. Anal. Calorim.* 2020, 139, 3577–3588.
84. Molana, M.; Dogonchi, A.; Armaghani, T.; Chamkha, A.J.; Ganji, D.; Tlili, I. Investigation of Hydrothermal Behavior of Fe<sub>3</sub>O<sub>4</sub>-H<sub>2</sub>O Nanofluid Natural Convection in a Novel Shape of Porous Cavity Subjected to Magnetic Field Dependent (MFD) Viscosity. *J. Energy Storage* 2020, 30, 101395.
85. Zafar, M.; Sakidin, H.; Dzulkarnain, I.; Afzal, F. Numerical Investigations of Nano-fluid Flow in Square Porous Cavity: Buongiorno’s Mathematical Model. In *Proceedings of the 6th International Conference on Fundamental and Applied Sciences, Kuching, Malaysia, 14–16 July 2020*; Springer: Berlin/Heidelberg, Germany, 2021; pp. 739–748.
86. Javadzadegan, A.; Joshaghani, M.; Moshfegh, A.; Akbari, O.A.; Afrouzi, H.H.; Toghraie, D. Accurate meso-scale simulation of mixed convective heat transfer in a porous media for a vented square with hot elliptic obstacle: An LBM approach. *Phys. A Stat. Mech. Appl.* 2020, 537, 122439.
87. Ram, D.; Bhandari, D.S.; Tripathi, D.; Sharma, K. Propagation of H1N1 virus through saliva movement in oesophagus: A mathematical model. *Eur. Phys. J. Plus* 2022, 137, 1–11.
88. Vijay, N.; Sharma, K. Heat and mass transfer study of ferrofluid flow between co-rotating stretchable disks with geothermal viscosity: HAM analysis. *Chin. J. Phys.* 2022, 78, 83–95.
89. Kumar, S.; Sharma, K. Impacts of Stefan Blowing on Reiner–Rivlin Fluid Flow Over Moving Rotating Disk with Chemical Reaction. *Arab. J. Sci. Eng.* 2022, 1–10.
90. Kefayati, G. Mixed convection of non-Newtonian nanofluid in an enclosure using Buongiorno’s mathematical model. *Int. J. Heat Mass Transf.* 2017, 108, 1481–1500.
91. Bondareva, N.S.; Sheremet, M.A.; Oztop, H.F.; Abu-Hamdeh, N. Entropy generation due to natural convection of a nanofluid in a partially open triangular cavity. *Adv. Powder Technol.* 2017, 28, 244–255.
92. Oliveski, R.D.C.; Macagnan, M.H.; Copetti, J.B. Entropy generation and natural convection in rectangular cavities. *Appl. Therm. Eng.* 2009, 29, 1417–1425.
93. Bejan, A. Second law analysis in heat transfer. *Energy* 1980, 5, 720–732.
94. Oztop, H.F.; Al-Salem, K. A review on entropy generation in natural and mixed convection heat transfer for energy systems. *Renew. Sustain. Energy Rev.* 2012, 16, 911–920.
95. Awais, M.; Ullah, N.; Ahmad, J.; Sikandar, F.; Ehsan, M.M.; Salehin, S.; Bhuiyan, A.A. Heat transfer and pressure drop performance of Nanofluid: A state-of- the-art review. *Int. J. Thermofluids* 2021, 9, 100065.
96. Liang, G.; Mudawar, I. Review of single-phase and two-phase nanofluid heat transfer in macro-channels and micro-channels. *Int. J. Heat Mass Transf.* 2019, 136, 324–354.
97. Sheikholeslami, M.; Rokni, H.B. Simulation of nanofluid heat transfer in presence of magnetic field: A review. *Int. J. Heat Mass Transf.* 2017, 115, 1203–1233.
98. Okonkwo, E.C.; Wole-Osho, I.; Almanassra, I.W.; Abdullatif, Y.M.; Al-Ansari, T. An updated review of nanofluids in various heat transfer devices. *J. Therm. Anal. Calorim.* 2021, 145, 2817–2872.
99. Kavva H.S., Dr. A.S. Hari Prasad, Amrutha H.P., Dr. Prashanth B., “Dispersion of passive & reactive solute in channel flows of a Kuvshiniski Viscoelastic fluid”, Scopus indexed Q2 Journal, *International Neurology Journal*, ISSN:2093-4777, E-ISSN:2093-6931, Schimago Citation Index Q2 SJR 2022 0.61, H-Index 32, Vol. 27, No. 4, pp. 751-762, Oct-Dec 2023. DOI: 10.5123/inj.2023.4.in81 <https://einj.net/index.php/INJ/article/view/245/196>

100.Amrutha H.P., Dr. A.S. Hari Prasad, Kavya H.S., Dr. Prashanth B., “Recent advances in the mathematical model developments of Convection processes in nano liquids occupying a tilted slot”, Scopus indexed Q2 Journal, International Neurology Journal, ISSN:2093-4777, E-ISSN:2093-6931, Schimago Citation Index Q2 SJR 2022 0.61, H-Index 32, Vol. 27, No. 4, pp. 726-733, Oct-Dec 2023. DOI: 10.5123/inj.2023.4.in <https://einj.net/index.php/INJ/article/view/243/194>