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Dispersion of passive & reactive solutes in channel flows using the concepts of unsteady convective diffusion of a passive solute in a Newtonian fluid flow thro' porous mediums

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Abstract

In this paper, a unsteady convective diffusion of a passive solute in a Newtonian fluid flow thro' porous mediums In this chapter, the brief conclusive remarks on the proposed research topic of "Unsteady convective diffusion of a passive solute in a Newtonian fluid flows thro' a porous medium" is presented along with the simulation results & its discussions. In conclusion, the design and experimental study on an inward flow reaction turbine using organic fluids and optimized concepts that has given us good empirical results.

Key Words:

Introduction

In this section, where the proposed objective is being solved, the unsteady convective diffusion of a passive solute in a Newtonian liquid flow thro' the porous mediums is presented along with different result in the field of fluid-based research that is being carried out & presented in this thesis report. Considering the last centuries, human beings had been accustomed to the utilization of the atmosphere and the waters of the earth as sinks for waste products and has relied on dispersion of these wastes in the fluid to reduce the concentration to tolerable levels. Increasing amounts of waste and increasing public resentment have brought the practice into dispute and the citizens in many countries are now determined to see higher standards of cleanliness in the air and water of the environment. The engineering and scientific professionals have the responsibilities of ensuring that similar type of standards were attained while allowing society to continue to enjoy the advantages of an industrial economy. To meet this obligation, engineers and other application-oriented scientists need to understand how dispersion occurs in fluids.

Overlying principle of operation

In principle, the problem of studying dispersion in the situations existing outside the physiological system is analogous to that within the corporeal system. The two differ in so far as the contaminant conditions are concerned. The conditions at the boundary of a physiological flow systems are far complex in nature than that existing in engineering problems. In technical literature on dispersion pertaining to laminar flows, the terms 'dispersion' and 'diffusion' will be not used always considering the clarity in the parameter selection process. Dictionary definitions e.g., the Oxford one, gives diffusion and dispersion in general use, as synonyms for each other. Finer shades of meaning than these are required in the science of fluid mechanics today. In literature, dispersion and diffusion are usually prefixed with 'convective' and 'molecular' respectively.

From a survey of relevant literature on dispersion of passive solute in channel flows, it becomes evident that unconsidered aspects need to be looked into the problems in the dispersion and these are: Modelling of Newtonian based liquids flows thro' porous medium, Modelling of Boundary conditions and Modelling of dispersion. The available approach for studying of the dispersion are: Small time dispersion (Lighthill 1966), Long-time dispersion (Taylor 1953, 1954, Aris 1956), All-time dispersion (Gill-Sankarasubramanian 1970).

Literature review

Literature based on Gill-Sankarasubramanian (GS) approach for Newtonian fluids is explained in this juncture is herewith mentioned as follows.

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Gill [1967] proposed a model involving a series expansion which is valid for all-time about the mean based concentrations. Gill [1967] generalized Taylor's work and his approach also led to overcome the limitations of Taylor's work. The expressions in the dispersion coefficients obtained by Gill reduces to Taylor's result in the limiting case of large Peclet No. & for small Peclet number, it reduces to Aris result where the axial molecular diffusion is considered to be more significant. Gill and Sankarasubramanian (GS) [1970] used the same series expansion to the studies in the non-steady convection type of diffusions for a flows thro' the circular tube and the correct solutions in the same showed that the diffusion coefficient varies with time. A generalized dispersion model for the average concentration Cm given by evolved group set of apparent dispersion coefficient K_i (i = 1,2,3,....), the principal components that is K_2. It is important to note that in the Taylor model, the apparent diffusions coefficient will be not dependent on time..

Annapurna and Gupta [1971], [1982] were the first to investigate dispersion using the GS model in electrically conducting Newtonian fluids. These authors presents an perfect analysis in the liquid solute dispersions using generalized model of Gill and Sankarasubramanian by considering electrically conducting fluid flow in the presense of constant magnetic field which is given along the transverse directions in between the two parallel plates. The authors displayed that using smaller values of Hartmann number M, the values in the dispersion coefficient show rapid fluctuations which decay with increase in M and for moderate and large values of M, these coefficients monotonically decrease with increase in M. It was shown by them that for every value in the Hartmann number the time-dependent coefficient increases up to some value and t attains a constant value (Taylor-Aris limit). The result of non-conducting fluids were obtained in the limit M to 0.

The exact analysis in the unsteady dispersions of a solutes was presented by Mandal and Mandal [1983]. They analyzed the dispersions of solutes for the convective radiating MHD flows of a fluids thro' vertical channel using linear variation in the wall temperatures. Mazumder and Dandapat [1984] considered the combination of forced & free convection flow to check the dispersions of solutes for the channel where the tempereture varied uniformly in the directions in the walls considering the axial directions. They obtained the diffusions coefficients after the slug being injected into the channel which was valid for all-time intervals. They also discovered that dispersion's coefficients will increases with Grashof number G provided both Grashof number and time intervals are small. The dispersion's coefficients will increase with 't' time slowly but after certain amount of time, the dispersion's coefficients increase and for large time it becomes equal to the asymptotic value. Further, considering the practical view point, the dispersion coefficient is independent of time for large Grashof number G. These earlier result will be valid considering both cooling and heating in the plate areas. Both these authors (Mazumder and Dandapat [1984]; Mandal and Mandal [1983]) had not shown the distribution of concentration and also they neglected the wall reaction.

Siddheshwar and Manjunath [2011] studied three-dimensional dispersion of a passive solute in a Hartmann flows when the case study of an electrical conducting type of newtonian liquid thro' the channel of rectangular cross section using generalized miscible dispersion theory of Gill and Sankarasubramanian [1970] was considered. More recently, Manjunath and Siddheshwar [2011] made a theoretical analysis of three-dimensional dispersion in couple-stress fluid flow using GS [1970] model as applied to flows following Doshi *et.al*. [1978]. Sankarasubramanian and Gill [1972] studied the solute dispersion in a Hagen Poiseuille flow in the Newtonian liquid considering the presence of wall reaction. The three important mechanisms in the presense of wall reaction case as against two in classical no-wall reaction have been reported. The wall reaction gives rise to exchange coefficient and the classical convective and diffusive coefficients and which are subjected to influence using the wall reactions. Annapurana and Gupta [1982] extended the studies of Gill and Sankarasubramanian [1972] to the hydro-magnetic case and showed which the Hartmann's No. has an effect identical to the wall reaction parameter.

Dutta *et.al*. [1985] found, using the GS model, the longitudinal dispersion coefficient of a solute for all time after injection in a solvent flowing under pulsatile condition in the presense of the periodic pressure gradient. In order to get few qualitative type of ideas considering the variation with time of the concentration of a solute injected in a pulsatile flow, an experiment was conducted in vitro in the thoracic aorta (where the flow is believed to be pulsatile) of two cocks. No attempts, how ever, have been achieved for comparing the result of all the experiments with the theoretical results derived by them in view of the distensibility of arterial walls. A study by Siddheshwar and Dulal Pal on dispersion in a plane Poiseuille flow between fluid permeable boundaries reveals that the assumed slip at the solvent permeable, solute-impermeable boundaries lead to enhanced dispersion. Gupta and Gupta [1972] studied the effect of homogeneous chemical reaction on unsteady convective diffusion that is validated for large time and they have identified the effects in the chemical reaction for the decreasing of the dispersion coefficient.

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Mandal *et.al*. [1983] using GS model, studied dispersion in a sparse packed type of porous media and showed its analogy to the MHD problem studied by Annapurna and Gupta [1979]. The earlier researchers have not evaluated the concentration distribution and also did not consider the pure-convection contribution. Rudraiah *et.al.* considered three-dimensional, long-time dispersion in flow through porous duct & because of this effects, the analogy between porous media flows and MHD are as good as obtained. A more general model of porous media (non-Darcy) was considered by Shivakumar *et.al.* and Rudraiah *et.al.* They studied dispersion using the generalized GS [1970] model. Using the GS [1970] model, the effects of inertia and boundary layer on dispersion a densely packed type of iso-tropic medium which is porous in nature & using a non-Darcy Forchheimer-Brinkman equation was studied by Rudraiah [1988]. Cvetkovic proposed a continuum model of solute transport in porous media.

Yates [1990] gave the analytical solution involving the relationship for the transport of dissolved substances in porous media with the dispersion which is distance-dependent. Shobha Devi [1999] used the GS approach in studying non-steady convection type of diffusions in the solutes through vertical and horizontal porous channels. An important feature in the works is the consideration of radiation and variable permeability. Siddheheshwar and Khalili presented a combinations of analytical and numerical solution for all-time analyzed results in the non-steady type of miscible dispersions in the chemically non-reactive solutes in a non-Darcy flow that accounts for Brinkman and Forchheimer effects in combination to the classical Darcy effects.

Mathematical analysis, modelling & formulation of equations

Effect of unsteady convective diffusions in a passive solute in a Newtonian fluid flows thro' the porous mediums is studied using a generalized miscible dispersion theory of Gill and Sankarasubramanian. Then, effects of porous medium on the velocities and thereby the convective and dispersion coefficients is discussed. The time-dependent dispersion coefficient and mean concentration distribution are computed and the result is represented graphically taking into consideration different value w.r.t. the porous parameter.

We analyze the mixed freed & forced convective laminar flows of a viscous in-compressible fluids b/w 2 parallel plates that are filled using a porous substances. The plate is separated by 2h distance and subjected to a uniform linear axial temperature variation. To facilitate our analysis, we utilize a rectangular type of coordinated system with the origin at the centre of channel. The x-axis aligns with the flows directions, while the y-axis is perpendicular w.r.t. the plates, as illustrated in Figure 3.

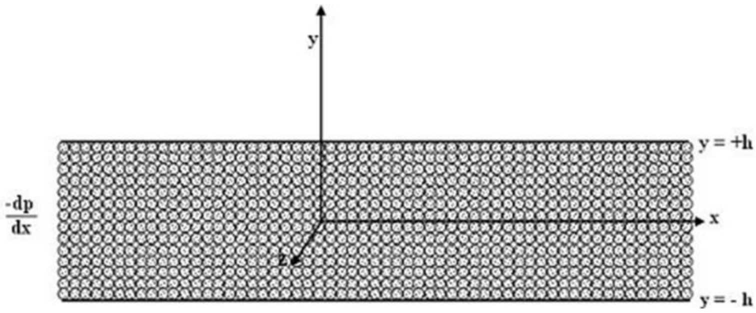

Fig. 1 : Fluid flows w.r.t. the 3 dimensional aspect point of view.

Figure 1 presents a three-dimensional depiction of the fluids flow. In the context of a constant fully developed type of laminar flows, the velocity (u) in the x direction is solely depends upon the y-coordinate. The equation of continuity ($\nabla \cdot q = 0$), where the velocity vector q has components (u, v, o), yields an integrated value of v as a constant. Because of the no-slip condition at the plates, this constant is zero throughout the fluid. Consequently, the equation of motions governing the fully developed parallel flows thro' the porous mediums are given in x & y directions as follows in the Table No. 1.

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C_w=V_w/V_a(%)	t_2 ±1 sec	t_2/t_1 ±0.001	μ_{exp} ±0.01 (millipoise)	ΔE_a ±0.02 (mev)	μ_{theo} ±0.01 (millipoise)	$\Delta\mu = \mu_{exp} - \mu_{theo}$ (millipoise) ±0.02
0	2107	1.364	8.79			
5	2180	1.412	9.15	1.07	8.92	0.23
10	2203	1.427	9.29	1.47	8.85	0.45
15	2228	1.443	9.45	1.92	8.78	0.67
20	2253	1.459	9.61	2.37	8.72	0.89
25	2278	1.475	9.76	2.78	8.67	1.08
30	2302	1.491	9.89	3.13	8.62	1.23
35	2328	1.508	10.05	3.56	8.57	1.47
40	2354	1.525	10.19	3.93	8.53	1.66
45	2378	1.540	10.33	4.29	8.49	1.84
50	2403	1.556	10.46	4.62	8.45	2.02
Water only	1544					

Table 1 : Quantitative results of the analysis, simulation & experimentation conducted for various type of elastic fluids

$$0 = -\frac{\partial p}{\partial x} - \frac{\mu}{k} u + \mu' \frac{d^2 u}{dy^2}$$

$$0 = -\frac{\partial p}{\partial y} - \rho g = \rho_0 [1 - \beta_1 (T - T_0)] g$$

where p, T, μ, k, ρ_g β_1 ρ_0 and T_o are the pressures, dynamic viscosity, permeabilities of porous mediums, densities of fluids, acceleration because of gravity field, the thermal type of expansion coefficients, temperatures, the temperature of reference states, densities of reference states, etc. The boundary based condition for the laminated flows are shown in the Fig. 2 as

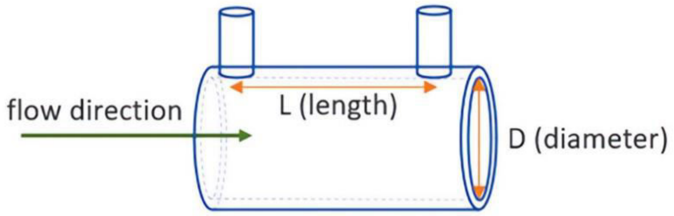

Fig. 2 : Flow direction thro' pipes with inlet & outlet for a length of *L*

$$u = 0$$

at

$$y = \pm h.$$

Under the assumption of a constant axial variation in temperatures along walls, then, the fluid temp could be expressed as follows:

$$T - T_0 = Nx + \varphi(y)$$

Here N is the steady temperature gradients in x-direction, and $\varphi(y)$ is a certain functions of the temperatures. Substitution of one equation in another equation and integration being carried out, one can see the new math model as

$$p = -\rho_0 g y + \rho_0 \beta_1 g N x y + \rho_0 g \beta_1 \int \varphi(y) dy + \psi(x)$$

where $\Psi(x)$ is the integration constant.

Elimination of the pressure values of p between one equation & the other equation, we get

$$\mu' \frac{d^2 u}{dy^2} - \frac{\mu}{k} u = \rho_0 \beta_1 g N y + \frac{d\psi}{dx}.$$

The mass balanced equations in the full developed uni-directional flow, pertaining to the solute concentration (C), along with the associated initial and boundary conditions, can be stated as follows::

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$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right],$$

$$C\ (0,\ x, y) = \begin{cases} C_0 & \text{for } |x| \le \frac{x_s}{2} \\ 0 & \text{for } |x| \ge \frac{x_s}{2} \end{cases}$$

$$\frac{\partial C}{\partial y}(t,\ x,\ h) = 0\ ,$$

$$\frac{\partial C}{\partial y}(t,\ x,\ 0) = 0\ ,$$

$$C(t,\ \infty,\ y) = \frac{\partial C}{\partial y}(t,\ \infty,\ y) = 0,$$

and

$$C(t,\ x,\ 0) = \text{finite}$$

where C_0 is the concentrations @ the initial levels of different input slug and x_s is is the lengths of the input slug. The above equation could be overwritten using the normal forms considering the initial distribution in the concentration levels of the slug parameters. Furthermore, the equation represents a balance of concentration flux and highlights the symmetry of C about the centerline. The subsequent centerline equation signifies that the solute does not extend to significantly distant locations from the source. All subsequent equations are built upon the assumption that the concentration along the centerline remains finite. Now, let us introduce the following dimensionless quantities:

$$\tau = \frac{t}{h^2/D},\quad X = \frac{x}{h\ Pe},\quad Y = \frac{y}{h},\quad \theta = \frac{C}{C_0},\quad P_x = -\frac{h^3}{\rho_0 v^2}\frac{d\psi}{dx},\quad U = \frac{uh}{vP_x},$$

where

$$Pe = \frac{u\,h}{D}$$

is the (Peclet number).

Using one equation in another equations, then one can get non-dimensional forms of equations as

$$\frac{d^2U}{dY^2} - \wedge\ \sigma^2\ U = -\wedge + \wedge\, G\, Y$$

where the parameter sigma, $\sigma^2 = \frac{h^2}{k}$ is the porous parameter

& the parameter $\Lambda = \frac{v^2\rho_0}{\mu'}$ is the Brinkmann number and

and $G = \frac{g\,\beta_1 h^4 N}{v^2 p_x}$ is the Grashof number.

Note that, since $P_x > 0$, the negative & positive values of N correspond to heating and cooling respectively along the walls of the channels. The related boundary conditions required for the solutions w.r.t. the above set of equations are …

$$U = 0$$

at

$$Y = \pm 1.$$

Solutions of the previous equations, subject to the parametric variations are modelled as

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$$U(Y) = \frac{1}{\sigma^2}\left[G\frac{\sinh(\sqrt{\wedge}\,\sigma Y)}{\sinh(\sqrt{\wedge}\,\sigma)} - \frac{\cosh(\sqrt{\wedge}\,\sigma Y)}{\cosh(\sqrt{\wedge}\,\sigma)} + 1 - GY \right].$$

Now, using one equation in another equations, we get their non-dimensional form as

$$\frac{\partial\theta}{\partial\tau} + U(Y)\frac{\partial\theta}{\partial X} = \frac{1}{Pe^2}\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2},$$

subject to parametric conditions given by

$$\theta_m(0, X, Y) = \begin{Bmatrix} 1, & |X| \le \frac{X_s}{2} \\ 0, & |X| \le \frac{X_s}{2} \end{Bmatrix},$$

$$\frac{\partial\theta}{\partial Y}(\tau, X, 1) = 0,$$

$$\frac{\partial\theta}{\partial Y}(\tau, X, 0) = 0,$$

$$\theta(\tau, \infty, Y) = \frac{\partial\theta}{\partial X}(\tau, \infty, Y) = 0,$$

$$\theta(\tau, X, 0) = \text{finite}.$$

Solutions of equation, subjected to conditions, now could be assumed in general form as given by (Gill and Sankarasubramanian [1970]).

$$\theta(\tau, X, Y) = f_0(\tau, Y)\theta_m(\tau, X) + f_1(\tau, Y)\frac{\partial\theta_m}{\partial X}(\tau, X) + f_2(\tau, Y)\frac{\partial^2\theta_m}{\partial X^2}(\tau, X),$$

where θ_m could be taken as the dimension-less type of c/s avg. concentrations & is modelled as

$$\theta_m(\tau, X) = \int_0^1 \theta\, dY$$

The previous equations signify that differences b/w θ and it's mean θ_m could be taken into account because of the diffusive type of contribution & the convection type of contributions, which would be based on the observation by Taylor [1953]. Integrating one equation w.r.t. *Y* taking the ranges of [0, 1] and utilizing the definitions for θ_m we get

$$\frac{\partial\theta_m}{\partial\tau} = \frac{1}{Pe^2}\frac{\partial^2\theta_m}{\partial X^2} - \frac{\partial}{\partial X}\int_0^1 U(Y)\,\theta\, dY$$

By incorporating one of the main equations into an auxiliary equation, we obtain the truncated form of the dispersion model proposed by Gill and Sankarasubramanian [1970], which can be expressed as given below.

$$\frac{\partial\theta_m}{\partial\tau} = K_0\theta_m + K_1\frac{\partial\theta_m}{\partial X} + K_2\frac{\partial^2\theta_m}{\partial X^2}$$

where k_i' could be assumed to be of the form as

$$K_i(\tau) = \frac{\delta_{i2}}{Pe^2} - \int_0^1 U(Y)\, f_{i-1}\, dY, \quad (i = 0, 1, 2)$$

Here, the parameter $f_{-1} = 0$ & δ_{i2} is called as the Kronecker' Delta.
By substitution of one equation in to other equation and utilizing the generalized dispersion model, the resulting equations for f_0, f_1, and f_2 can be expressed as follows:

$$\frac{\partial f_k}{\partial \tau} = \frac{\partial^2 f_k}{\partial Y^2} - U(Y) f_{k-1} + \frac{1}{Pe^2} f_{k-2} - \sum_{i=0}^{k} K_i f_{k-i}, \ (k = 0, 1, 2).$$

where

$$f_{-1} = f_{-2} = 0$$

It can be noted here that for the evaluation of the parameter K_k's, one has to study the effects of f_k's. Then, we proceed to solve for the parametric equations considering different parameter f_k subjected to different boundary conditions as follows.

$$f_k (\tau, 0) = \text{finite},$$

$$\frac{\partial f_k}{\partial Y}(\tau, 1) = 0$$

$$\frac{\partial f_k}{\partial Y}(\tau, 0) = 0$$

$$\int_0^1 f_k (\tau, Y) dY = \delta_{k0}, \qquad (k = 0, 1, 2).$$

The earlier equations have been derived by substitution of one equation into another equation. So, substituting $k = 0$ in equation, then the 2nd order differential equations for the f_0 could be visulaized in the form of

$$\frac{\partial f_0}{\partial \tau} = \frac{\partial^2 f_0}{\partial Y^2}$$

We assume the solution of the equation in the idealized forms as

$$f_0 (\tau, Y) = f_{01}(Y) + f_{02}(\tau, Y),$$

This gives the following two differential equations

$$\frac{d^2 f_{01}}{dY^2} = 0$$

and

$$\frac{\partial f_{02}}{\partial \tau} = \frac{\partial^2 f_{02}}{\partial Y^2}$$

The boundary based conditions & the initial condition can be obtained as

$$\frac{d f_{01}}{dY} = 0, \ at \ Y = 0,1$$

$$\frac{d f_{02}}{dY} = 0, \ at \ Y = 0,1,$$

$$\int_0^1 f_{01} dY = 1. f_{01} = 1$$

Solving of the equation subjected w.r.t. different boundary based condition, researcher could get the constrained equation. Then, solutions of one of the equation subjected to different other boundary conditions could be given by

$$f_{02}(\tau, Y) = \sum_{j=0}^{\infty} A_j \exp[-\mu_j^2 \tau] \cos(\mu_j Y),$$

where

$$\mu_j = j\pi$$

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will be the equation roots given by

$$\sin(\mu_j)=0$$

& and A_j's will be modelled as

$$A_j = -\int_0^1 f_{01} \cos \mu_j (Y)\, dY$$

Since $f_{01} = 1$ & $A_j's$ vanish on utilizing the condition in the boundary constraints. Thus, $f_{02} = 0$ and hence f_0 are modelled by $f_{02} = 1$ and could be re-written as f_{02}.
Putting the values of $i = 1$ for the previous equation, and noting that $f_0 = 0$, one could get the new equation as

$$K_1 = -\int_0^1 U(Y)\, dY.$$

Substituting one equation in another equation and simplifying we get

$$K_1 = -\frac{1}{\sigma^2}\left[\frac{G\cosh(\sqrt{\wedge}\sigma)}{\sqrt{\wedge}\sigma\sinh(\sqrt{\wedge}\sigma)} - \frac{\sinh(\sqrt{\wedge}\sigma)}{\sqrt{\wedge}\sigma\cosh(\sqrt{\wedge}\sigma)} - \frac{G}{\sqrt{\wedge}\sigma\sinh(\sqrt{\wedge}\sigma)} + 1 - \frac{G}{2}\right]$$

It is obvious from equation that K_1 could be identified utilizing the avg. velocity & therefore, the name - convection coefficient for K_1 could be visualized.

Results & Discussions

Effect of non-steady convection diffusions in the passive type of solutes for the Newtonian fluid flows thro' the porous mediums are studied using a generalized miscible dispersion theory of Gill and Sankarasubramanian. The analysis follows closely the research works of Gill and Sankarasubramanian and Gill (1973). We now discuss the results obtained in our research work as shown in the Fig. 3.

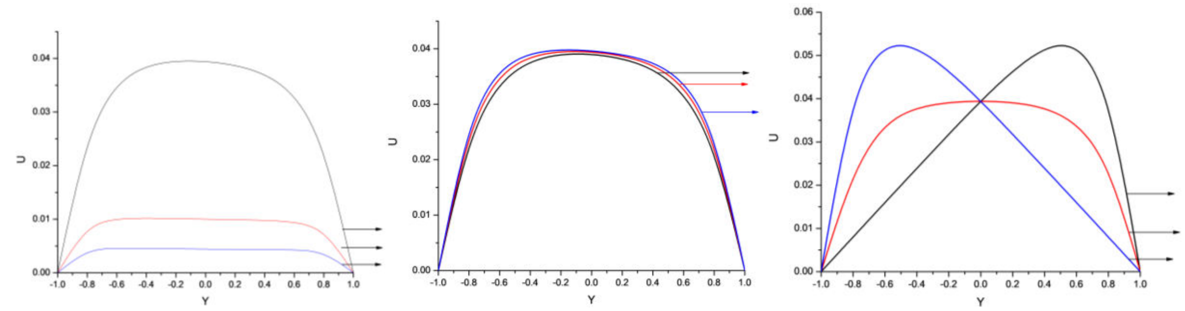

Fig. 3 : Plots of filter velocity distributions for different value of σ, Λ & *G* = 0.555 / Plots of filter velocity distributions for different value of σ, Λ, *G* = 0.655 / Plots of filter velocity distributions for different value of Λ = 5, σ = 1 & G

Here, the parameters of sigma with varying values are given by

$\sigma = 5 \quad \Lambda = 0.8$

$\sigma = 10 \quad \Lambda = 1.0$

$\sigma = 15 \quad \Lambda = 1.2$

for the purpose of simulations.

Conclusions

In conclusion, the study of non-steady convection diffusions of a passive type of solutes for the Newtonian fluid flows thro' the porous mediums will yields several key findings.

Firstly, it is evident that the unsteadiness of the flow play a vital roles in transport of the solute. The time-dependent variations in the flow concentration distribution & the velocities that will affect the overall solute transport characteristics in the system. Understanding the transient behaviours for the solute concentrations will be very much crucial for a no. of variety of applications, viz., the environmental remediation, chemical engineering processes, and groundwater contamination studies.

Secondly, the convective nature of the solute transport, driven by the fluid flow, enhances the dispersion and mixing of the solute within the porous medium. This convective contribution, along with the diffusive transport mechanism, governs the overall solute transport behavior. It is essential to consider both convective and diffusive effects when analyzing the solute transporting of the porous medias under the influence of non-steady flowing conditions.

Additionally, the characteristic features of the porous medium, such as its porosity, permeability, and tortuosity, greatly influence the solute transport behavior. These properties dictate the flow field within the porous medium, impacting the convective transport and diffusion rates. Therefore, a comprehensive understanding of the porous medium's properties is essential for accurately predicting the solute transport behavior.

Furthermore, the analysis of non-steady convection type of diffusions in the Newtonian fluids flow through a porous medium provides insights into the designing & the optimizing of various engineering processes. By considering the solute transport dynamics, engineers and authors could develop many powerful strategies for pollutant remediation, groundwater management, and other related applications.

In a nut-shelled closings, the study of non-steady convection type of diffusions w.r.t. the passive type of elastic fluid flows w.r.t. the Newtonian fluid flows thro' the porous mediums contributes to our understanding of solute transport phenomena and helps in developing practical solutions for real-world problems involving porous media systems. The figures gives the following information's.

- plots for the filter velocity distributions as a f/n for the non-dimensional transverse coordinate Y and porous parameter σ.
- plots for the filter velocity distributions as a f/n for the non-dimensional transverse coordinate Y and Brinkmann number Λ.
- plots for the filter velocity distributions as a f/n for the non-dimensional transverse coordinate Y and Grashof number G. $G < 0$ and $G > 0$ correspond respectively to cooling and heating along the plates.
- plots w.r.t. the convection type of coefficients $-K_1$ versus σ for various value of Λ. Clearly - K_1 increase with the increasing in the Λ as σ increases.
- plots w.r.t. the convection type of coefficients $-K_1$ versus σ for various value of G. Clearly - K_1 deccreases with the increasing in the G as σ increases.
- From plots, it could be inferred where, the parameter K_2 will decrease with the increasing of the parameter σ.
- Also, we also infer that from the simulation results, the K_2 parameter will increase with increasing of the G.
- After discussing the dispersive mechanism, we will now proceed to make observations on the parameter theta θ_m.

The plots illustrate the relationship between the mean concentration distribution (θ_m theta_m) and the dimensionless time (tau τ) for various values of sigma. It is important to note that in this scenario, the convective and diffusive coefficients exhibit time-dependence, making the obtained values of θ_m theta_m versus tau highly accurate. Upon examining the figure, we observe that concentrations outside the initial slug (viz., $X > X_s$) are noticeably lower compared to points within the slug. Additionally, we notice that theta_m decreases as the flow rate (G) increases, both for points inside and outside the input slug. The simulation plots depict typical Gaussian distributions, represented by the error function solution of the unsteady convective-diffusive system with time-dependent coefficients. Furthermore, we observe that θ_m theta_m increases with an increase in sigma σ, as depicted in the figure.

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